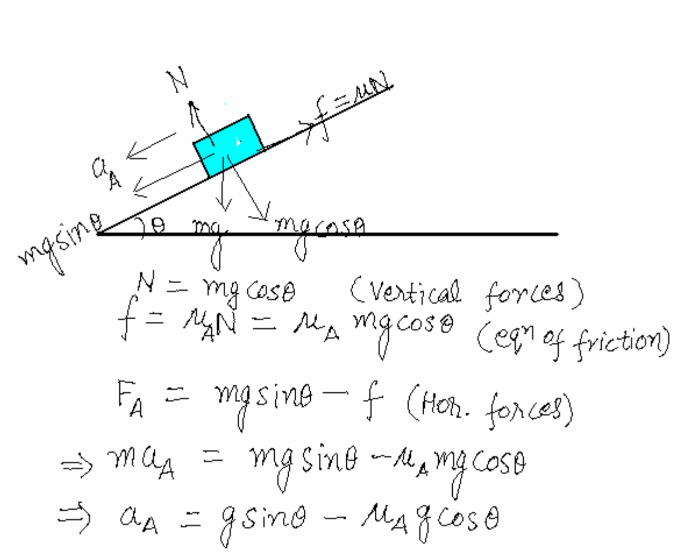
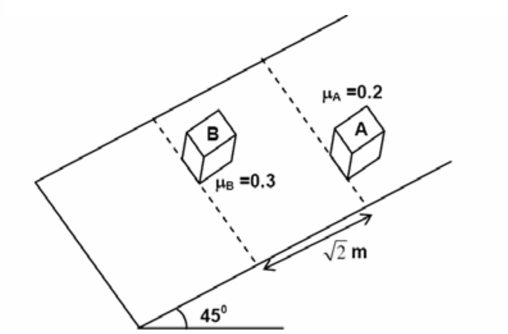
Problem: 1 FORCE,FRICTION & KINEMATICS

Two identical blocks A and B are placed on a rough inclined plane of inclination 45^0. The coefficient of friction between block A and incline is 0.2 and that of between B and incline is 0.3. The initial separation between the two blocks is $\sqrt{2}$ m. The two blocks are released from rest, then find
(a) The time after which front faces of both blocks come in same line. and
(b) The distance moved by each block for attaining above position.

Solution:

a)





Using free body diagram given above, acceleration of block A is given by

$$a_A = g\sin 45° - 0.2g\cos 45° = 4\sqrt{2}\ m/s^2$$

Similarly for acceleration of block B is given by

$$a_B = g \sin 45° - 0.3\, g \cos 45° = \frac{7}{2}\sqrt{2}\ \text{m/s}^2$$

Acceleration of block A with respect to block B is given by

$$a_{AB} = a_A - a_B$$

$$a_{AB} = 0.5\sqrt{2}\ \text{m/s}^2$$

Relative distance to be covered can be represented as

$$s_{AB} = \frac{1}{2} a_{AB} t^2$$

As we know that, Relative distance to be covered is given as $\sqrt{2}$ m, so

$$t^2 = \frac{2\sqrt{2}}{0.5\sqrt{2}} = 4$$

$$t = 2 \text{ sec.}$$

Hence time after which front faces of both blocks come in same line is 2 seconds.

a) **Distance travelled by block B is**

$$s_B = \frac{1}{2} a_B t^2 = 7\sqrt{2}\ \text{m}$$

Distance travelled by block A is

$$s_A = \frac{1}{2} a_A t^2 = 8\sqrt{2}\ \text{m}$$

Problem: 2 WORK AND ENERGY

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0 X 10^3 N/m, as shown in Fig. The spring is compressed 2.0 cm and is then released from rest.

(a) Calculate the speed of the block as it passes through the equilibrium position *x = 0* if the surface is frictionless.

Solution:

In this situation, the block starts with

Initial velocity $v_i = 0$
at initial position $x_i = -2.0$ cm,

Now we want to find final velocity v_f at final position $x_f = 0$.

We use following Equation to find the work done by the spring with

$x_{\max} = x_i = -2.0$ cm $= -2.0 \times 10^{-2}$ m:

$$Ws = \frac{1}{2}kx_{max}^2$$

$= \frac{1}{2}(1.0 \times 10^3 \text{ N/m})(-2.0 \times 10^{-2} \text{ m})^2$

= 0.20 J

Using the work–kinetic energy theorem with $v_i = 0$, we obtain the change in kinetic energy of the block due to the work done on it by the spring:

$$Ws = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$0.20\,\text{J} = \frac{1}{2}(1.6\,kg)v_f^2 - \frac{1}{2}(1.6\,kg)0^2$$

$v_f^2 = \frac{2 \times 0.20\,\text{J}}{(1.6\,kg)}$ = 0.25 m^2/s^2

V_f = .5 m/s

(b) Calculate the speed of the block as it passes through the equilibrium position if a constant frictional force of 4.0 N retards its motion from the moment it is released.

Solution:

Certainly, the answer has to be less than what we found in part (a) because the frictional force retards the motion.
We use following Equation to calculate the kinetic energy lost because of friction and add this negative value to the kinetic energy found in the absence of friction. The kinetic energy lost due to friction is

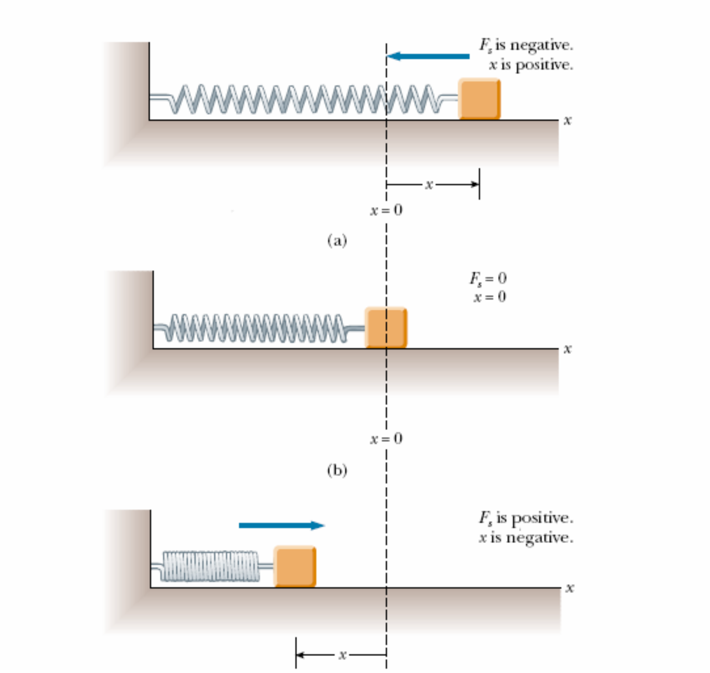
$$\blacktriangle k_f = -f_k d$$
$$= -(4.0\,\text{N})(2.0 \times 10^{-2}\,\text{m})$$
$$= -0.080\,\text{J}$$

In part (a), the final kinetic energy without this loss was found to be 0.20 J. Therefore, the final kinetic energy in the presence of friction is

$$K_f = 0.20 - 0.80$$
$$= 0.12J$$
$$= \frac{1}{2} m v_f^2$$

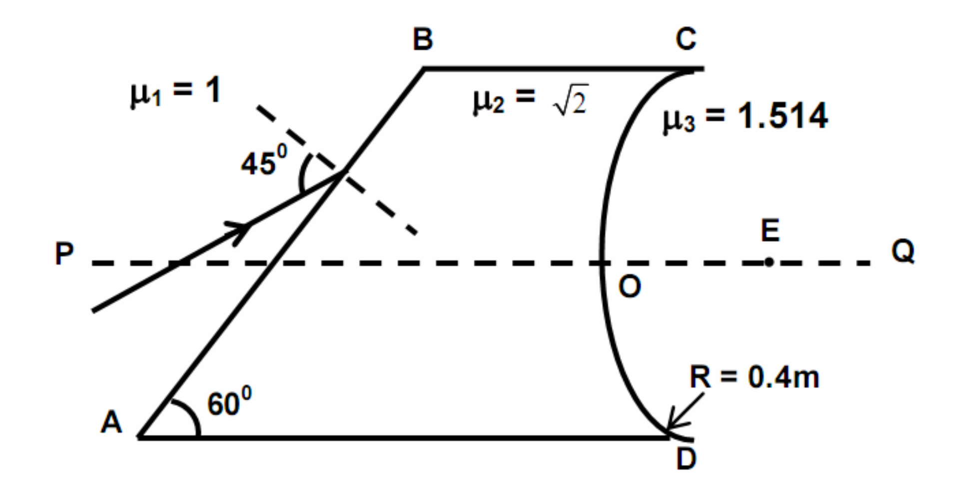
$v_f^2 = \frac{0.24J}{1.6kg}$ = 0.15 m^2/s^2

V_f = 0.39 m/s
As expected, this value is somewhat less than the 0.50 m/s we found in part (a). If the frictional force were greater, then the value we obtained as our answer would have been even smaller.

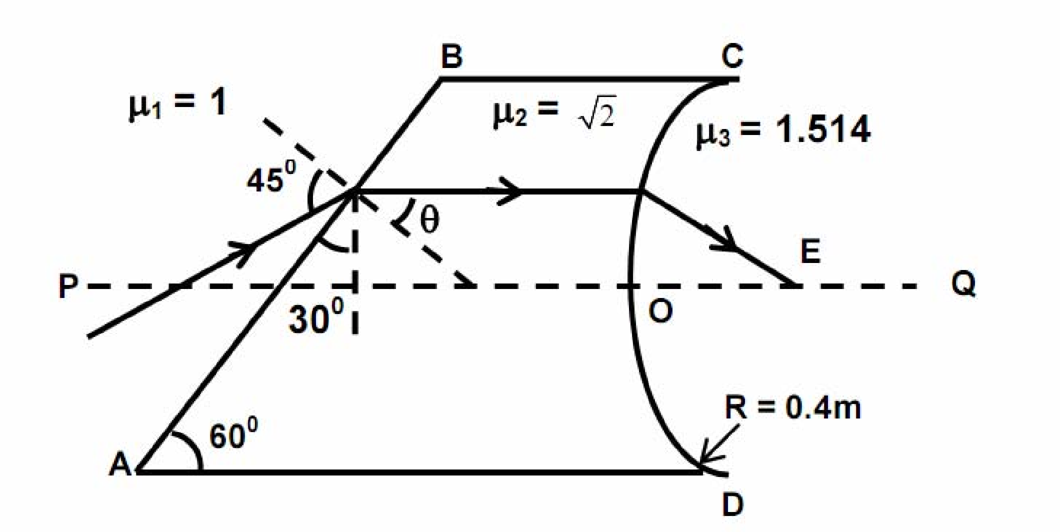


Problem: 3 REFRACTION

A light ray is incident on an irregular shaped slab of refractive index 2 at an angle of 45° with the normal on the incline face as shown in the figure. The ray finally emerges from the curved surface in the medium of the refractive index μ = 1.514 and passes through point E. If the radius of curved surface is equal to 0.4 m, find the distance OE correct up to two decimal places.



Solution:



As we know by Snell's law

$\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2$

Using Snell's law

$\mu_1 \sin 45° = \mu_2 \sin \theta$

$1.\ (\frac{1}{\sqrt{2}}) = = \sqrt{2} \sin \theta$

$\Theta = \sin^{-1}(\frac{1}{2})$

$\theta = 30°$.

i.e. ray moves parallel to axis.

Using the lens maker's formula; we have

$$\frac{\mu_3}{OE} - \frac{\mu_2}{\infty} = \frac{(\mu_3 - \mu_2)}{R}$$

Where
μ_3 is the refractive index of the medium containing emergent ray = 1.514

μ_2 is the refractive index of the given slab = 2
μ_1 is the refractive index of the medium containing incident ray = 1
OE is the distance of final image of the ray from the pole (O) = ?
Since ray moves parallel to X-axis distance of object from pole (O) = ∞
R is radius of curvature of curved surface = 0.4m

Substituting above values in the given equation; we get

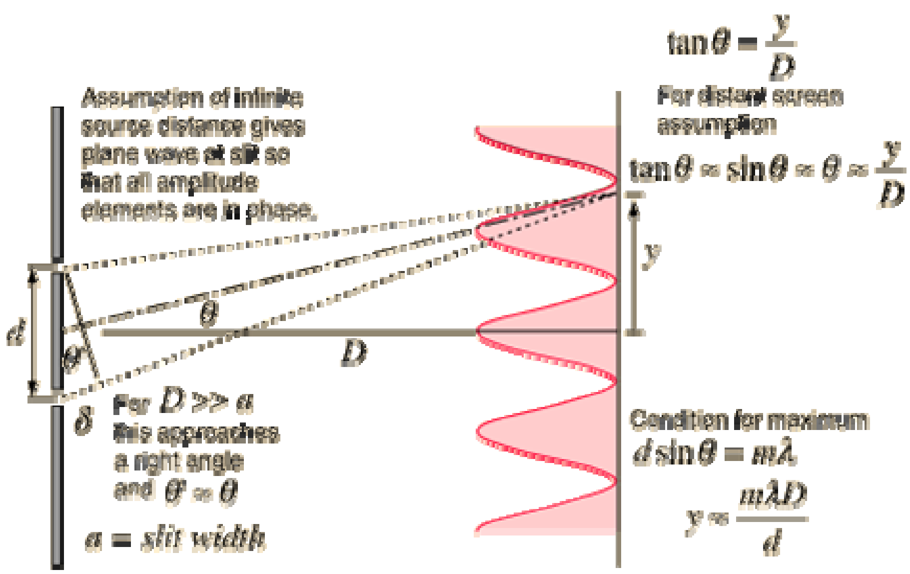
$$\frac{1}{OE} = \frac{1.514-2}{.4}$$

OE = 6.056 m ≈ 6.06 m

Problem: 4 WAVE OPTICS

In a Young's double slit experiment light consisting of two wavelengths λ_1 = 500 nm and λ_2 = 700 nm is incident normally on the slits.
Find the distance from the central maxima where the maxims due to two wavelengths coincide for the first time after central maxima. (Given $\frac{D}{d} = 1000$) where D is the distance between the slits and the screen and d is the separation between the slits.

Solution:



The distance from the central maxima to the n^{th} maxima of first wave is given by

$y_1 = (n_1 D\lambda_1)/d$

The distance from the central maxima to the n^{th} maxima of second wave is given by

$y_2 = (n_2 D\lambda_2)/d$

Where;
n_1, n_2 are integers.
D is the distance between the slits and the screen.

d is the separation between the slits.

The central maxima and maxims due to two wavelengths coincide for the first time after central maxima if we have

$y_1 = y_2$

□ $(n_1 D\lambda_1)/d = (n_2 D\lambda_2)/d$

□ $n_1 = (7/5)\ n_2$

Since n_1 and n_2 are integers;
For the first location,
$n_2 = 5$, $n_1 = 7$
(Note: if we will substitute any other value like n_1 =1,2,3 or 4 then we will get n2 as fraction which is not required)

□ $y = 7 \times 1000 \times 5 \times 10^{-7}$

$= 35 \times 10^{-4}$

= 3.5 mm.

Problem: 5 RAY OPTICS

A point object is moving with velocity 0.01 m/s on principal axis towards a convex lens of focal length 0.3m. When object is at a distance of 0.4 m from the lens, find
(a) rate of change of position of the image, and
(b) rate of change of lateral magnification of image.

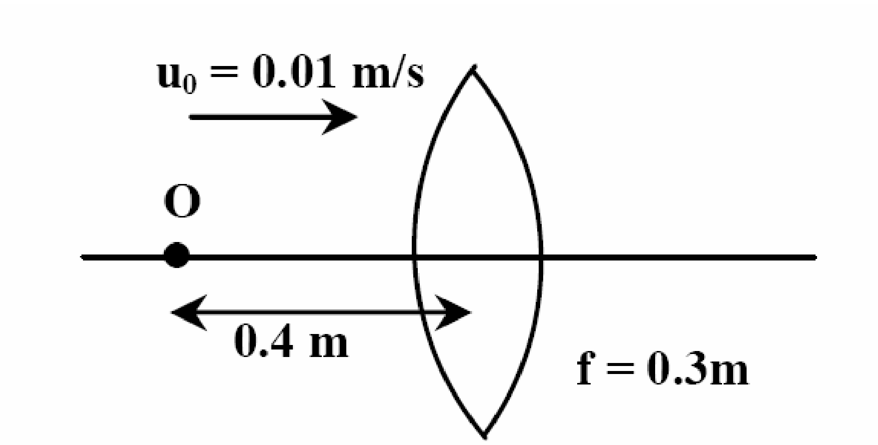
Solution:

(a) The lens equation is shown below:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

For a lens of focal length **f = .3m = 30 cm**
An object distance of **u = .4m = 40 cm**
Will produce an image at **v =?**

As per the situation given in the problem corresponding figure can be drawn as



Where;

u_0= $\frac{du}{dt}$ = .01m/s =1cm/s is the given velocity of object.

Now, on differentiating lens equation we have

$$\frac{-1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt} \quad \text{.........(i)}$$

On substituting values of u and f in lens equation, we get

□ $\frac{1}{30} = \frac{1}{v} - \frac{1}{-40}$

□ $\frac{1}{v} = \frac{1}{30} - \frac{1}{40}$

□ $\frac{1}{v} = \frac{1}{120}$

□ $v = 120\ cm$

Now, on substituting values of u, $\frac{du}{dt}$ and v in equation (i), we get

□ $\frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt}$

□ $\frac{dv}{dt} = \frac{120^2}{40^2}.1$

□ $\frac{dv}{dt} = 3^2.1 = \frac{9cm}{s} = .09m/s$

(b) Lateral magnification of image is given by

□ $m - \frac{dv}{du}$

□ $m = \frac{\frac{dv}{dt}}{\frac{du}{dt}}$

□ $m = \frac{\frac{v^2}{u^2}\frac{du}{dt}}{\frac{du}{dt}}$

□ $m = \frac{v^2}{u^2}$

as we know;

$\frac{v}{u} = 1 - \frac{v}{f}$, we have

□ $m = \left(1 - \frac{v}{f}\right)^2$

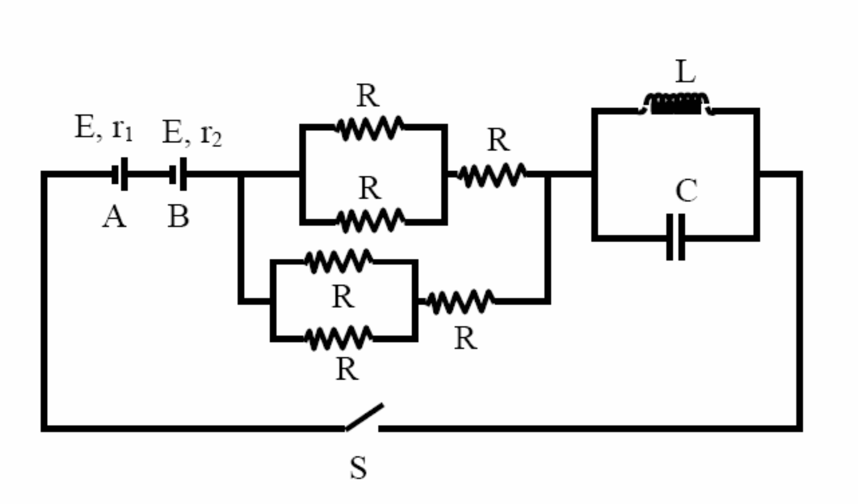
On differentiating **m,** we get rate of change of lateral magnification as

$$\frac{dm}{dt} = -\frac{2}{f}\left(1 - \frac{v}{f}\right)\frac{dv}{dt}$$

$$= \frac{-2}{0.3}\left(1 - \frac{120}{30}\right) \times 0.09 = 1.8\ s^{-1}$$

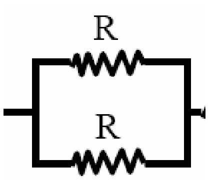
Problem: 6 ELECTRICITY

The two batteries A and B, connected in given circuit, have equal e.m.f. E and internal resistance r1 and r2 respectively (r1 > r2). The switch S is closed at t = 0. After long time it was found that terminal potential difference across the battery A is zero. Find the value of R.



Solution:

First of all we will calculate the equivalent resistance.



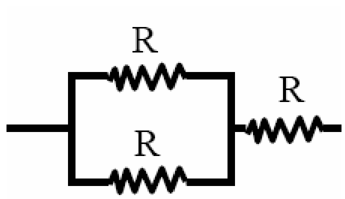
Equivalent resistance of above stretch is given by

$(R \ \& \ R \ in \ parallel)$

$$= \frac{1}{\frac{1}{R}+\frac{1}{R}} = \frac{R}{2}$$

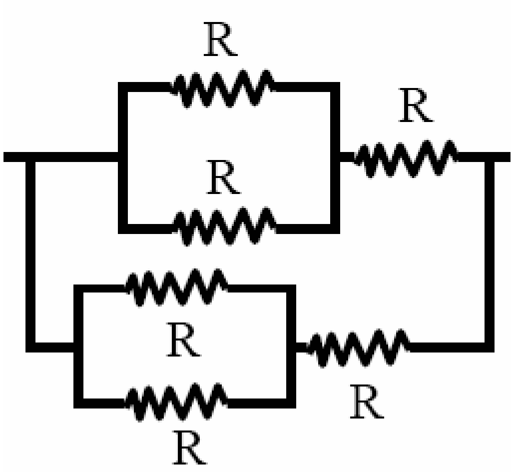
So now the equivalent resistance of following stretch is

$(\frac{R}{2} \ \& \ R \ in \ series) = \frac{R}{2} + R =$



$\frac{3R}{2}$

Hence we can find the equivalent resistance of the given circuit as follows



$(\frac{3R}{2} \& \frac{3R}{2}\ in\ parallel)$

$$R_{eq} = \frac{1}{\frac{2}{3R}+\frac{2}{3R}}$$

$$R_{eq} = \frac{3R}{4}$$

After long time it will reach till steady state. Since average voltage across capacitor and inductor for D.C. sources will be zero at steady state. The net effect of resistance will be only due to resistors i.e. R_{eq}, r_1 and r_2.

So, the net resistance offered is

(R_{eq}, r_1 and r_2 in series)

R_{net} = R_{eq} + r_1 + r_2

The net e.m.f. offered is due to (E & E in series) =E + E = 2E

So, current through the circuit can be given by

$$I = \frac{2E}{(R_{eq} + r_1 + r_2)} = \frac{2E}{(r_1 + r_2 + \frac{3R}{4})}$$

P.D. across the battery A $= E - Ir_1 = 0$

$\Rightarrow I = E/r_1$

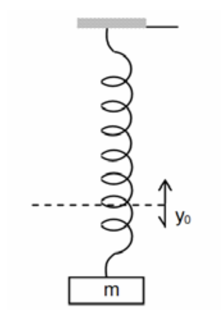
From (i) and (ii),

$$R = \frac{4(r_1 - r_2)}{3}$$

Hence above equation is required value of R.

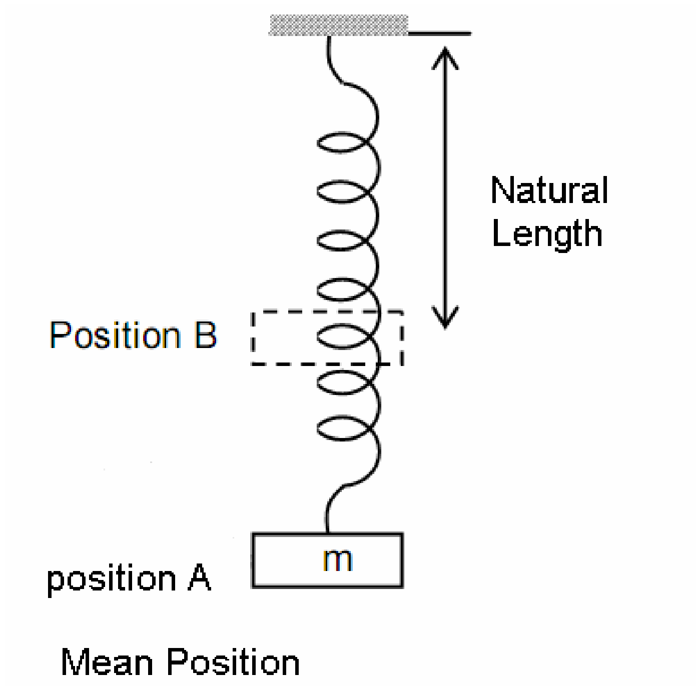
Problem: 7 SHM(Simple Harmonic Motion)

A small body attached to one end of a vertically hanging spring is performing SHM(Simple Harmonic Motion) about it's mean position with angular frequency(ω) and amplitude(a). If at a height y* from the mean position the body gets detached from the spring, calculate the value of y* so that the height H attained by the mass is maximum. The body does not interact with the spring during it's subsequent motion after detachment. (a ω^2> g).



Solution:

At position B as the potential energy of the spring will be zero, the total energy (Gravitational potential energy + Kinetic energy) of the block at this point will be maximum and therefore if the block gets detached at this point, it will rise to maximum height.



F = kY*

Where F is gravitational force = mg Newton

K is the spring constant. (N/m)

Y* is the maximum extension.

So,

Mg = kY*

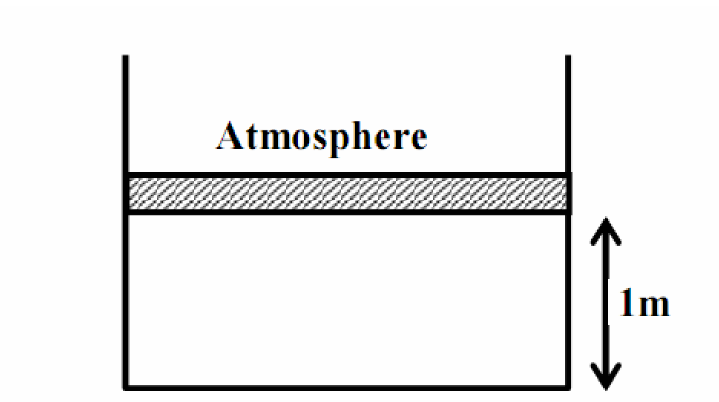
- ⇨ Y*= mg/k
- ⇨ Y*=g/(k/m)
- ⇨ Y* = g/ω^2 (since $\sqrt{k/m} = \omega^2$)
- ⇨ Y* = $g/\omega^2 < a$ (since it is given that $a\omega^2 > g$)

Hence Y*= g/ω^2 is the required value.

Problem: 7 THERMODYNAMICS

An ideal diatomic gas is enclosed in an insulated chamber at temperature 300K. The chamber is closed by a freely movable massless piston, whose initial height from the base is 1m. Now the gas is heated such that its temperature becomes 400 K at constant pressure. Find the new height of the piston from the base.

If the gas is compressed to initial position such that no exchange of heat takes place, find the final temperature of the gas.



Solution:

Since the gas is heated such that its temperature becomes 400 K at constant pressure. The process is **Isobaric**.

Hence;

Volume/time = V/T is constant i.e.

$V_1/ T_1= V_2/ T_2$

As we know,

Volume = (Area)x(Length)

$V_1 = A \times 1 = A$

$V_2 = A \times h = Ah$

$T_1 = 300k$

$T_2 = 400k$

So,

$V_1 / T_1 = V_2 / T_2$

- ⇨ $A / 300k = Ah / 400k$
- ⇨ $h = 4/3$ m

Now, the gas is compressed to initial position such that no exchange of heat takes place . The process involved is **Adiabatic**.

As we know in this process,

$TV^{\gamma-1} = \text{Constant}$

$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$

$V_2 = A \times h = A \times 4/3$

$V_2 = A \times 1 = A$

(since it is diatomic gas we have $\gamma = 7/5$)

$T_2 = 400k$

$T_3 = ?$

$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$

- ⇨ $400 (A \times 4/3)^{(7/5-1)} = T_3(A)^{(7/5-1)}$
- ⇨ $T_3 = 400 (4/3)^{(2/5)}$
- ⇨ $T_3 = 448.78$ kelvin